Effect of CEV on option prices under jump-diffusion dynamics with stochastic volatility: A finite element method approach

1st Abdelilah Jraifi
MISCOM, ENSA Safi,
Cadi Ayyad University, Morocco
jraifi.abdelilah@gmail.com

2nd Aziz Darouichi
L2IS, FST Marrakech,
Cadi Ayyad University, Morocco
az.darouichi@uca.ma

3rd Ilias Elmouki
LERMA, EMI Rabat,
Mohammed V University, Morocco
i.elmouki@gmail.com

***Abstract*—In this paper, we propose a stochastic volatility jump-diffusion model with a constant elasticity of variance β in the variance equation. This model describes dynamics of the asset price S_t and its variance V_t, based on two stochastic differential equations (SDEs) with Poisson jumps. The resolution of these two SDEs, is essential to find the sought European option price which depends on both main two variables S_t and V_t through a partial integro-differential equation (PIDE). The existence and uniqueness of solution of this PIDE in a weighted Sobolev space, are established based on a variational formulation of the considered problem which we solve using the finite element method (FEM). Spatial differential operators are discretized using $P1$ elements, while the time stepping is performed using an explicit Euler scheme. Finally, we provide some numerical results based on the FEM to show the effect of different values of β on the option prices.**

***Index Terms*—Jump-diffusion model, Option pricing, Partial integro-differential equation, Finite elements method**

I. Introduction

In finance literature, it is common to represent the uncertainty of the economy by a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathcal{P})$ where $\mathcal{F}_t$ is the filtration of available information at time t, and $\mathcal{P}$ is the real probability measure. All processes that we shall consider in this section will be defined in this space. An asset price model with stochastic volatility, in which stochastic volatility follows the CEV model has been defined in [3] as follows

$$\left\{ \begin{array}{lcl} dS_t &=& \mu S_t dt + S_t V_t dW_t \\ dV_t &=& pdt + \eta V_t^{\beta} dW_t^{'} \end{array} \right. \tag{1}$$

where μS_t is the drift term, V_t is the volatility, ηV_t^{β} is the volatility term, while p, η and β are non-negative constants. W_t and $W_t^{'}$ are defined as two Brownian motions with $\langle W_t, W_t^{'} \rangle = \rho$ (i.e. ρ is the correlation factor).

The numerical resolution of such problems, has been provided in [1] using the finite element method when there is no jump and also, where the authors have considered the jump-diffusion model (Bates's model [4]) with a stochastic volatility which follows the Heston's model [19] for options pricing. On the other hand, Eraker and Polson in [13], extended Bate's work in 2003, by incorporating jumps in the volatility equation also. Their model is given by

$$\left\{ \begin{array}{lcl} dS_t &=& \mu S_t dt + S_t \sqrt{V_t} dW_t + S_{t^-} Y_t dN_t^s \\ dV_t &=& pdt + \eta \sqrt{V_t} dW_t^{'} + Z_t dN_t^V \end{array} \right. \tag{2}$$

where N_t^s and N_t^V are independent Poisson processes.

Broadie and Kaya [6] have performed exact method for Eraker's et al model for the evaluation of an European call under the stock index $S\&P500$ by the Monte-Carlo method. In this paper, we would like to generalize the volatility term of the volatility in (2). In fact, we consider V_t^{β} instead of $\sqrt{V_t}$ in the volatility equation, and we propose the finite element method for comparing the obtained results in [6] and improving the convergence of the RMS error in dimension 2.

The rest of this paper is organized as follows: In section 2., we present the constant elasticity of variance with jumps (CEVJ) model for option pricing problem. In section 3., we introduce a weighted Sobolev space and the variational formulation for the considered problem. In section 4., we provide and compare the obtained numerical results by the two methods mentioned before, namely, the execution time and the RMS error for different values of β. Finally, we conclude our work in section 5.

II. Model Descriptions

First, we assume there is a risk-neutral probability measure $\mathcal{Q}$, for more details, refer to [26] and references therein. We consider an European derivative on S_t, denoted by $w(t, S_t, V_t)$ with expiration date T and payoff function h, and by r; the interest rate. Its price at a time t, will depend on time t, on the price of the underlying asset S_t, and on the volatility V_t. It is given by the risk-neutral expected discounted payoff

$$w(t,s,v) = \mathbb{E}^{\mathcal{Q}}\left[e^{-r(T-t)} h(S_T) / S_t = s, V_t = v\right], \tag{3}$$

In the risk-neutral world, the considered model are governed by the following dynamics:

$$\left\{ \begin{array}{lcl} dS_t &=& (r - \lambda^s m) S_t dt + S_t \sqrt{V_t} dW_t + S_{t^-} Y_t dN_t^s \\ dV_t &=& (p - R(t,s,v)\eta V_t^{\beta}) dt + \eta V_t^{\beta} dW_t^{'} + Z_t dN_t^V \end{array} \right. \tag{4}$$

where S_t, V_t, p, η and β are defined above in (2). Moreover, r is the risk-free interest rate, N_t^s and N_t^v are independent Poisson processes with constant intensities λ^s and λ^v respectively. Y_t is the jump size of the asset price return with density $\phi_Y(y)$ and $E(Y_t) := m < \infty$, while Z_t is the jump size of the volatility with density $\phi_Z(z)$.
In addition, we have

$$R(t,s,v) = \rho\,\frac{\mu - r}{v_t} + \sqrt{1-\rho^2}\;\theta^*(t,s,v),$$

where ρ is the correlation factor and $\theta^*(t,s,v)$ is an arbitrary function, for more details, see page 45 in [14]. We note that N_t^s and N_t^v are independent of standard Brownian motions W_t and $W_t^{'}$.
Since every compound Poisson process can be represented as an integral form of a Poisson random measure (see page 82 "section 3.2" in [7], and equation (3.8) "section 3.3" in [7]), we have

$$\int_0^t Z_s^* dN_s = M_t = \int_0^t \int_{\mathbb{R}} zN(ds,dz)$$

where $N(dt,dz)$ is a Poisson random measure of the process $M_t = \sum_{i=1}^{N_t} Z_i^*$ with intensity measure $\lambda\phi(dz)dt$.
Then, the dynamics on the right hand side of (4) can be rewritten as follows

$$\begin{cases} dS_t &= (r-\lambda^s m)S_t dt + S_t\sqrt{V_t}dW_t + \int_{\mathbb{R}} S_{t^-}yN^s(dt,dy) \\ dV_t &= (p - R(t,s,v)\eta V_t^{\beta})dt + \eta V_t^{\beta} dW_t^{'} + \int_{\mathbb{R}} zN^v(dt,dz) \end{cases} \tag{5}$$

N^s is a Poisson random measure of the process $\sum_{i=1}^{N_t} Y_i$ with intensity measure $\nu^s(dy)dt = \lambda^s\phi_Y(y)dt$, and N^v is a Poisson random measure of the process $\sum_{j=1}^{N_t} Z_j$ with intensity measure $\nu^v(dz)dt = \lambda^v\phi_Z(z)dt$.
In the following, we present a theoretical formulation of the considered problem (5), and we provide a proof of the existence and uniqueness of solution in this general case.

A. General formulation

Let us consider a financial asset whose price $\{S_t, t \geq 0\}$ is given by the following jump-diffusion stochastic differential equation (see page 10 in [8]):

$$dS_t = F(t,S_t,V_t)dt + G(t,S_t,V_t)dW_t + \int_{\mathbb{R}^n} \gamma(t,S(t^-),z)\widetilde{N}^S(dt,dz), \tag{6}$$

where $S_0 = s_0 \in \mathbb{R}^n$, $F : [0,T]\times\mathbb{R}^n\times\mathbb{R}^d \longrightarrow \mathbb{R}^n$, is a drift term, W_t is a m-dimensional Brownian motion, and $G : [0,T]\times\mathbb{R}^n\times\mathbb{R}^d \longrightarrow M_{n\times m}(\mathbb{R})$, is the stochastic volatility. V_t is given by the following stochastic differential equation:

$$dV_t = a(t,S_t,V_t)dt + b(t,S_t,V_t)dW_t^{'} + \int_{\mathbb{R}^d} \chi(t,V(t^-),\varsigma)\widetilde{N}^V(dt,d\varsigma), \tag{7}$$

with $V_0 = v_0 \in \mathbb{R}^d$. The simplest models (see [9]) take a constant volatility, but these models are generally not smooth enough to match real price. The operator $a : [0,T]\times\mathbb{R}^n\times\mathbb{R}^d \longrightarrow \mathbb{R}^d$, is a drift term of volatility, $b : [0,T]\times\mathbb{R}^n\times\mathbb{R}^d \longrightarrow M_{d\times m}(\mathbb{R})$, is the volatility of volatility, and $W_t^{'}$ is a linear combination of W_t and an independent Brownian motion B_t defined by : $W_t = \rho W_t^{'} + \sqrt{1-\rho^2}B_t$, where the correlation ρ is some constant in $[-1,1]$.

$$\begin{aligned} \widetilde{N}^S(dt,dz) &= (\widetilde{N}_1^S(dt,dz),....,\widetilde{N}_{l_S}^S(dt,dz)) \\ &= \left(N_1^S(dt,dz_1) - \nu_1^S(dz_1)dt, ..., N_{l_S}^S(dt,dz_{l_S}) - \nu_{l_S}^S(dz_{l_S})dt\right) \\ \widetilde{N}^V(dt,d\varsigma) &= (\widetilde{N}_1^V(dt,d\varsigma),....,\widetilde{N}_{l_V}^V(dt,d\varsigma)) \\ &= \left(N_1^V(dt,d\varsigma_1) - \nu_1^V(d\varsigma_1)dt, ..., N_{l_V}^V(dt,d\varsigma_{l_V}) - \nu_{l_V}^V(d\varsigma_{l_V})dt\right), \end{aligned}$$

are respectively l_S, l_V independent compensated Poisson random measures, independents of $W()$. For each $k \in \{1,2,...,l_S\}$ and $k' \in \{1,2,...,l_V\}$ we have: $\widetilde{N}_k^S(dt,dz) = N_k^S(dt,dz) - \nu_k^S(dz)dt$ and $\widetilde{N}_{k'}^V(dt,d\varsigma) = N_{k'}^S(dt,d\varsigma) - \nu_{k'}^V(d\varsigma)dt$, where ν_k^S, $\nu_{k'}^V$ are the Lévy measures (intensity measures) of the Poisson random measures $N_k^S(.,.)$ and $N_{k'}^V(.,.)$ respectively, see (Theorem 1.7, page 3 in [8], Appendix). We mention that $\{N_k^S\}$ and $\{N_{k'}^V\}$ are independent Poisson random measures with Lévy measures ν^S, ν^V respectively, for all $k \in \{1,2,...,l_S\}$ and $k' \in \{1,2,...,l_V\}$.
Moreover, we assume that the jump processes $\widetilde{N}^S$ and $\widetilde{N}^V$ are independent of standard Brownian motions W_t and $W_t^{'}$.
$\gamma : [0,T]\times\mathbb{R}^n\times\mathbb{R}^n \longrightarrow M_{n\times l_S}(\mathbb{R})$, $(t,s,z) \overset{\gamma}{\longmapsto} \gamma(t,s,z)$, and $\chi : [0,T]\times\mathbb{R}^d\times\mathbb{R}^d \longrightarrow M_{d\times l_V}(\mathbb{R})$, $(t,v,\varsigma) \overset{\chi}{\longmapsto} \chi(t,v,\varsigma)$, are respectively $n\times l_S$ and $d\times l_V$ matrix of measurable real valued functions which are adapted processes such that the integrals exist. For a detailed presentation of jump-diffusion model, we refer to [8].

$$\gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1l_S} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2l_S} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nl_S} \end{pmatrix},$$

$$\chi = \begin{pmatrix} \chi_{11} & \chi_{12} & \cdots & \chi_{1l_V} \\ \chi_{21} & \chi_{22} & \cdots & \chi_{2l_V} \\ \vdots & \vdots & \ddots & \vdots \\ \chi_{d1} & \chi_{d2} & \cdots & \chi_{dl_V} \end{pmatrix}.$$

$\gamma^{(k)} := (\gamma_{1k},\gamma_{2k},....,\gamma_{nk})$, $\chi^{(k')} := (\chi_{1k'},\chi_{2k'},....,\chi_{dk'})$ for $k \in \{1,2,...,l_S\}$ and $k' \in \{1,2,...,l_V\}$.
Note that each column $\gamma^{(k)}$, $\chi^{(k')}$ respectively of the $n\times l_S$ and $d\times l_V$ matrix $\gamma = [\gamma_{ij}]$, depends on z and ς only through the k^{th}, k'^{th} coordinate $z_k, \varsigma_{k'}$, i.e $\gamma^{(k)}(t,s,z) = \gamma^{(k)}(t,s,z_k)$. $\chi^{(k')}(t,v,\varsigma) = \chi^{(k')}(t,v,\varsigma_{k'})$, $z \in \mathbb{R}^{l_S}$, $\varsigma \in \mathbb{R}^{l_V}$.
If the following assumptions are satisfied:

- The functions F, G, a, b, γ, χ, are measurable.

For every $t \in [0,T]$, $s,s' \in \mathbb{R}^n$, and $v,v' \in \mathbb{R}^d$, and there exists two constants K_1, K_2 such that,

- $\|F(t,s,v) - F(t,s',v')\| + \|G(t,s,v) - G(t,s',v')\| + \int_{\mathbb{R}^n} \|\gamma(t,s,z) - \gamma(t,s',z)\|\widetilde{N}^S(dt,dz) \leq K_1(\|s-s'\| + \|v-v'\|)$

- $\|F(t,s,v)\|^2 + \|G(t,s,v)\|^2 + \int_{\mathbb{R}^n} \|\gamma(t,s,z)\|^2 \widetilde{N}^S(dt,dz) \leq K_1^2(1+\|s\|^2)$
- $\|a(t,s,v) - a(t,s',v')\| + \|b(t,s,v) - b(t,s',v')\| + \int_{\mathbb{R}^n} \|\chi(t,v,\varsigma) - \chi(t,v',\varsigma)\| \widetilde{N}^V(dt,d\varsigma) \leq K_2(\|s-s'\| + \|v-v'\|)$
- $\|a(t,s,v)\|^2 + \|b(t,s,v)\|^2 + \int_{\mathbb{R}^n} \|\chi(t,v,\varsigma)\|^2 \widetilde{N}^V(dt,d\varsigma) \leq K_2^2(1+\|v\|^2)$
- S_0, V_0 are square-integrable.

Then, the solution of the system (6)-(7) is unique, for more details see ([10], [11], [12]).
We note that in our special case, we have here $F(t,S_t,V_t) = r - \lambda^s m$,
$G(t,S_t,V_t) = S_t\sqrt{V_t}$, $a(t,S_t,V_t) = p - R(t,s,v)\eta V_t^\beta$ and $b(t,S_t,V_t) = \eta V_t^\beta$.

In the following, we formulate the variational problem associated to (5).

III. Variational problem

In weighted Sobolev spaces, Bensoussan and Lions [5], considered that the value of the function w defined in (3), can be characterized as the solution of the following PIDE.

$$\begin{cases} \frac{\partial w}{\partial t}(t,s,v) - \mathcal{L}w(t,s,v) = 0 & \forall\, t \in [0,T], s \in \mathbb{R}, v \in \mathbb{R}^+ \\ w(T,s,v) = h(s) & s \in \mathbb{R}, v \in \mathbb{R}^+ \end{cases} \tag{8}$$

with

$$\begin{aligned} \mathcal{L}w = {} & \frac{1}{2}v^2 s^2 \frac{\partial^2 w}{\partial s^2} + \rho\eta v^{\beta+1} s \frac{\partial^2 w}{\partial s \partial v} + \frac{1}{2}\eta^{2^{2\beta}} \frac{\partial^2 w}{\partial v^2} \\ & +(r-\lambda^s m)s\frac{\partial w}{\partial s} + (p - R(t,s,v)\eta v^\beta)\frac{\partial w}{\partial v} - \\ & (r-\lambda^s m)w + \int_{\mathbf{R}} \Big[u(s+sy,v) - u(s) - syu'_s(s,v)\Big]\nu^s(dz) \\ & + \int_{\mathbf{R}} \Big[u(s,v+z) - u(s) - zu'_v(s,v)\Big]\nu^v(dz) \end{aligned} \tag{9}$$

Then, we use the variational formulation in order to solve the partial integro-differential equation $(PIDE)$ by the finite element method (FEM).
For this, let be $U = [0,+\infty[\times]a,b[$ with $0 < a < b < +\infty$ is a domain in $\mathbb{R}^2$. Let $\delta = (s,v)$ be a vector in U, with the Euclidean norm defined by the formula $|\delta| = \sqrt{s^2+v^2}$.
We introduce some weighted Sobolev spaces, $\mathrm{L}^2_\alpha(U)$ is a space of measurable functions u and 2^{th} integrable for the measure $e^{-\alpha|\delta|}d\delta$ on U where $\alpha > 0$ and $d\delta = dsdv$. The variational formulation of (8) consists of finding a continuous function u defined on the time interval $[0,T]$ with value in the following weighted Sobolev space (see [1])
$\mathrm{W}^{1,2}_{\alpha,1}([0.T] \times U) \equiv \mathrm{W}^{1,2}_{\alpha,1}$ the space of functions u in $\mathrm{L}^2\Big(0,T;\mathrm{W}^{1,2}_\alpha(U)\Big)$ such that $\frac{\partial u}{\partial t} \in \mathrm{L}^2\Big(0,T;\mathrm{L}^2_\alpha(U)\Big)$.
$\mathrm{W}^{1,2}_\alpha(U)$ is the space of functions u in $\mathrm{L}^2_\alpha(U)$ such that $\frac{\partial u}{\partial x_i} \in \mathrm{L}^2_\alpha(U)$, equipped with the norm

$$\|u\|_{\mathrm{W}^{1,2}_\alpha(U)} := \|u\|_\alpha = \left(\int_U |u|^2 e^{-\alpha|\delta|}d\delta + \int_U \sum_{i=1}^2 \Big|\frac{\partial u}{\partial x_i}\Big|^2 e^{-\alpha|\delta|}d\delta\right)^{1/2}. \tag{10}$$

Multiplying the PIDE in (8) with a test function $u' \in \mathcal{D}(U)$, we obtain the associated variational formulation, with $u(t,s,v) = w(T-t,s,v)$:

$$\begin{cases} (\frac{\partial u}{\partial t}, u') + (\mathcal{L}u, u') = 0 & \forall\, t \in [0,T] \text{ and } (s,v) \in U \\ (u(0,.,.), u') = (h, u') & \forall\,(s,v) \in U \end{cases} \tag{11}$$

using the Green's formula and the Dirichlet boundary conditions, we get:

$$\begin{cases} (\frac{\partial u}{\partial t}, u') - a(u,u') = 0 \;\forall\, t \in [0,T] \text{ and } (s,v) \in U \\ (u(0,.,.), u') = (h, u') \quad \forall\,(s,v) \in U \end{cases} \tag{12}$$

with

$$\begin{aligned} a(u,u') = {} & \frac{1}{2}\int_U v^2 s^2 \frac{\partial u}{\partial s}\frac{\partial u'}{\partial s} e^{-\alpha|\delta|}d\delta + \int_U v^2 s \frac{\partial u}{\partial s} u' e^{-\alpha|\delta|}d\delta \\ & - \int_U (r-\lambda^s m)s\frac{\partial u}{\partial s}u' e^{-\alpha|\delta|}d\delta + \int_U \frac{\eta^2}{2}v^{2\beta}\frac{\partial u}{\partial v}\frac{\partial u'}{\partial v}e^{-\alpha|\delta|}d\delta \\ & + \int_U \beta\eta^2 v^{2\beta-1}\frac{\partial u}{\partial v}u' e^{-\alpha|\delta|}d\delta - \int_U (p - \eta v^\beta R)\frac{\partial u}{\partial v}u' e^{-\alpha|\delta|}d\delta \\ & + \int_U \rho\eta v^{\beta+1} s \frac{\partial u}{\partial v}\frac{\partial u'}{\partial s}e^{-\alpha|\delta|}d\delta + \int_U \rho\eta v^{\beta+1}\frac{\partial u}{\partial v}u' e^{-\alpha|\delta|}d\delta \\ & + \int_U (r-\lambda^s m)uu' e^{-\alpha|\delta|}d\delta - \int_U \frac{\alpha s^3 v^2}{2|\delta|}\frac{\partial u}{\partial s}u' e^{-\alpha|\delta|}d\delta \\ & - \int_U \rho\alpha\eta\frac{sv^{\beta+1}}{|\delta|}\frac{\partial u}{\partial s}u' e^{-\alpha|\delta|}d\delta - \int_U \frac{\alpha\eta^2 v^{2\beta+1}}{2|\delta|}\frac{\partial u}{\partial v}u' e^{-\alpha|\delta|}d\delta \\ & - \int_U \left(\int_{\mathbf{R}} \Big[u(s+sy,v) - u(s) - sy\frac{u}{\partial s}\Big]\nu^s(dy)\right)u' e^{-\alpha|\delta|}d\delta \\ & - \int_U \left(\int_{\mathbf{R}} \Big[u(s,v+z) - u(v) - z\frac{u}{\partial v}\Big]\nu^v(dz)\right)u' e^{-\alpha|\delta|}d\delta \end{aligned} \tag{13}$$

where

$$(\mathcal{L}u, u') = -a(u,u')$$

For $h \in \mathrm{W}^{1,2}_\alpha \cap \mathrm{L}^\infty$, the variational problem (12) admits a unique solution in $\mathrm{W}^{1,2}_{\alpha,1} \cap \mathrm{L}^\infty$. This solution has the probabilistic representation (3), for more details see ([1], [2]).
In the next section, we present and discuss some numerical results.

IV. Numerical results

With the growing complexity of models and derivatives, the numerical methods associated with assessing financial options became an important field of research over the last decade. In the present section, we will implement a European call using the associated PIDE (12) to the CEVJ model (5) for option pricing. We will present the results of the simulations given by finite elements method using FreeFem++ software. The numerical experiments were performed on a Sony Vaio Laptop with an Intel® Pentium® CPU P6100@ 2.00 GHZ processor and 4 Go of RAM, running Windows 7 (64 bits).

We consider a European call for the $S\&P500$ stock index whose true value is equal to 6.8619 for this purpose. The parameters used in our numerical experiments are $K = 100$, $p = 0.00346$, $\rho = -0.82$, $R_{moy} = 3.14$, $\eta = 0.05$, $r = 3.19\%$, $T = 1\ year$.
Then, we will solve and compare the resolution approach of variational problem for the pricing of the considered European option by using the finite elements approximation in space, and an explicit Euler discretization in time.
For the numerical simulation, we consider the problem (12) on a bounded domain $\Omega = (S_{min}, S_{max}) \times (v_{min}, v_{max})$, where (v_{min}, v_{max}) does not contain zero. The corresponding variational problem is then given by

$$\left\{ \begin{array}{rcl} (\frac{\partial u}{\partial t}, u') - a(u,u') & = & 0, \ \forall\, t \in [0,T], \ \ (s,v) \in \Omega \\ \big(u(0,.,.), u'\big) & = & \big(h, u'\big) \qquad \forall\, (s,v) \in \Omega \end{array} \right. \tag{14}$$

where $a(u, u')$ is given by (13). The resolution of this problem using FreeFem++ with $P1$ finite elements, provides the following numerical results as illustrated in Table I and Table II.

TABLE I
COMPUTING TIMES OF FEM IN SECOND UNIT, ASSOCIATED TO CEV VALUES UTILIZED FOR EACH NUMBER OF TIME STEPS

No of time steps	Comput times (sec)	Weight of space α	Power of volatility β
10	2.0256	0.30217	2/5
20	12.7682	0.82002	1/3
40	18.8287	2.1362469	2/7
80	37.3328	2.2957453	1/4
160	58.1834	2.3424291	2/9

In Table I, we present different values of CEV, namely β, associated to each number of time steps. This number of iterations has been changed in a way to show the time when it has been observed there is a reduction or minimization of the difference between the values of estimated and true option prices based on different statistical factors as it will be discussed hereafter.

TABLE II
BIAS, MEAN SQUARED ERROR (MSE) AND THE VALUES OF THE OPTION PRICE ASSOCIATED TO EACH CEV VALUE

Power of volatility β	Bias	MSE	Estimated price of option	True price of option
2/5	0.5023	0.327447	7.3084	6.8061
1/3	0.4451	0.001161	6.9512	
2/7	0.2167	0.000196	7.0228	
1/4	0.1246	0.000004	6.9307	
2/9	0.0835	0.0000002	6.8896	

In Table II, we show the numerical values of the estimated option price for different values of CEV. We can deduce from this table that as more the value of CEV is small, as more the value of the bias and the mean squared error (MSE) become small, while the estimated option price becomes closer to the true option price when β is chosen small.

V. CONCLUSION

The aim of this paper, is to show the effect of CEV on the pricing of the European option under jump-diffusion model with stochastic volatility. The study has been based on the formulation of a variational problem resolved using the finite elements method (FEM) for some values of β or more precisely when it is strictly smaller than 1/2. We concluded that the obtained values of the option price are closer to the true market values of the European option exercised under the stock index $S\&P500$ on March 2, 2014.

REFERENCES

[1] R. Aboulaich, F. Baghery, A. Jraifi, Option pricing for a stochastic volatility jump-diffusion model, International Journal of Mathematics and Statistics 13, p. 1-19, 2013.
[2] R. Aboulaich, F. Baghery, A. Jraifi, Numerical approximation for options pricing of a stochastic volatility jump-diffusion model, Int. J. Applied. Math. Stat 50, p. 69-82, 2013.
[3] R. Aboulaich, L. Hadji, A. Jraifi, Option pricing with constant elasticity of variance (CEV) model, Applied Mathematical Sciences 7, p. 5443-5456, 2013.
[4] D.S. Bates, Jumps and stochastic volatility : exchange rate processes implicit in deutsche mark option, The Review of financial Studies 9, p. 69-107, 1996.
[5] A. Bensoussan, J.L. Lions, Contrôle Impulsionnel et Inéquations Quasi-Variationnelles, Dunod, Paris, 1980.
[6] M. Broadie, O. Kaya, Exact simulation of stochastic volatility and other affine jump diffusion processes, Operations Research 54, p. 217-231, 2006.
[7] R. Cont, P. Tankov, Financial modeling with jump processes, edition of Chapman & Hall/CRC Financial Mathematics, Boca Raton, FL, Volume 1, 2004.
[8] B. Øksendal, A. Sulem, Applied stochastic control of jump diffusions (2nd Edition), Universitext, Springer-Verlag, New York, 2007.
[9] X. Zhang, Numerical analysis of American option pricing in a jump-diffusion model, Mathematics of operation research 22, p. 668-690, 1997.
[10] I.I.Gikhman, A.V. Skorokhod, Stochastic differential equations, Springer Verlag, New York, 1972.
[11] J. Jacod, A.N. Shiryaev, Limit theorems for stochastic processes, Grundlehren der Mathematischne Wissenschaften, Springer Verlag, New York, 1987.
[12] M. Johannes, N. Polson, J. Stroud, Filtering of stochastic differential equations with jumps, Working paper, Columbia University 11, p. 1-44, 2002.
[13] B. Eraker, M. Johannes, N. Polson, The impact of jumps in equity index volatility and returns, Journal of Finance 58, p. 1269-1300, 2008.
[14] J.P. Fouque, G. Papanicolaou, K.R. Sircar, Derivatives in Financial Markets with stochastic Volatility, Cambridge University Press, 2000.
[15] K.O. Friedrichs, The identity of weak and strong extensions of differential operators, Rans, Amer, Math, Soc 55, p. 132-151, 1987.
[16] J.W. Gao, Optimal Portfolios for DC Pension Plans under a CEV Model, Insurance : Mathematics and Economics 44, p. 479-490, 2009a.
[17] J.W. Gao, Optimal Investment Strategy for Annuity Contracts under the Constant Elasticity of Variance (CEV) Model, Insurance : Mathematics and Economics 45, p. 9-18, 2009b.
[18] J.W. Gao, An extended CEV model and the Legendre transform-dual-asymptotic solutions for annuity contracts, Insurance : Mathematics and Economics 46, p. 511-530, 2010.
[19] S. Heston, A Closed-Form solution for Options with stochastic volatility with applications to bond and currency options, Review of Financial Studies 6, p. 327-343, 1993.
[20] Y.L. Hsu, T.I. Lin, C.F. Lee, Constant Elasticity of Variance (CEV) Option Pricing Model, Integration and Detailed Derivation, Mathematics and Computer in Simulation 79, p. 60-71, 2008.
[21] J. Hull, A. White, The pricing of options on assets with stochastic volatilities, Journal of Finance 42, p. 281-300, 1989.
[22] J.L. Lions, E. Magenes, Méthode Numérique pour les Equations aux Dérivées Partielles en Finance, Addison-Wesley Reading, Massachusetts, 1994.

[23] A.E. Lindsay, D.R. Brecher, Simulation of the CEV process and the local martingale property, Mathematics and Computers in Simulation 82, p. 868-878, 2012.
[24] J.D. Macbeth, L.J. Merville, Tests of the Black-Scholes and Cox Call Option Valuation Models, Journal of Finance 35, p. 285-300, 1980.
[25] P. Protter, Stochastic Integration and Differential Equations, Second Edition, Springer-Verlag, 2003.
[26] H. Geman, N. El Karoui, J.C. Rochet, Changes of Numeraire, Changes of Probability Measure and Option Pricing, Journal of Applied Probability 32, p. 443-458, 1995.
[27] J. Xiao, H. Zhai, C. Qin, H. Zhai, C. Qin, The Constant Elasticity of Variance (CEV) Model and the Legendre Transform-Dual Solution for Annuity Contracts, Insurance : Mathematics and Economics 40, p. 302-310, 2007.